A MIMO Cautious Self-Tuning Controller

This paper presents a simple linear model based self-tuning controller (STC) for MIMO systems. The controller, namely the cautious self-tuning controller (CSTC), features enhanced robustness characteristics in the case of unmodeled nonlinearities and/or dynamics. The controller structure is that of a linear quadratic, minimizing a single-step cost function, and with time varying input penalization. The CSTC can be applied to open-loop stable and minimum-phase systems. Application to a simulated distillation column demonstrates its effectiveness.

Apostolos V. Papadoulis
Department of Chemistry
and Chemical Engineering
Michigan Technological University
Houghton, MI 49931

Spyros A. Svoronos
Department of Chemical Engineering
University of Florida
Gainesville, FL 32611

Introduction

Adaptive controllers are generally based on linear models. Most chemical processes, however, are inherently nonlinear. In addition, extra nonlinearities are introduced by nonlinear sensors and control valves. The existing linear-model-based, adaptive algorithms face serious problems when applied to highly nonlinear systems. One might observe large sustained oscillations, failure of the algorithm to drive the system to the setpoint, and even instability. Such incidents have been reported by Song et al. (1984), Gustafsson (1984), and Papadoulis et al. (1987) for the minimum variance self-tuning regulator (STR).

Self-tuning controllers (Clarke and Gawthrop, 1975) involving input penalization have been proved to be more robust than the STR (Gawthrop and Lim, 1982) and are perhaps the most industrially accepted adaptive controllers (Dumont, 1986). The major drawback these controllers face is the requirement to know, *a priori*, weights that result in a closed-loop stable system. For open-loop stable systems, stability and robustness can be achieved with heavy penalization of the control variable, but this results in sluggish performance. Pole/zero placement adaptive controllers with fixed poles and fast performance specifications are faced with similar problems (Papadoulis, 1987). Another alternative is the approach taken by generalized predictive control (GPC) algorithms (e.g., Ydstie and Liu, 1984; Ydstie et al., 1985; Clarke et al., 1987a, 1987b). Robustness is enhanced by aiming to reach the set points (or achieve another performance measure) at a time (horizon) greater than the minimum dictated by time delays and zeroes outside the unit circle. With constant horizons, this approach has the disadvantage that large horizons might be necessary to ensure robustness, thus leading to sluggish performance.

Correspondence concerning this paper should be addressed to S. A. Svoronos.

To circumvent these problems, algorithms involving input penalization with time-varying weights automatically adjusted on-line, have been presented. Allidina and Hughs (1980) calculate the weights on-line as the solution to pole placement related, Diophantine equations. Latawiec and Chyra (1983) utilize on-line stability criteria to adjust the weights. Toivonen (1983) interprets the input weight as the Langrange multiplier of a control variance constrained optimal control problem. These algorithms (all developed for SISO systems) require a large amount of on-line computations which may become prohibitive in the multivariable case.

A different approach for some nonlinear systems is the use of nonlinear adaptive controllers. Promising algorithms have been proposed by Gustafsson (1984), Golden and Ydstie (1985), and Agarwal and Seborg (1987). But these controllers require that the nonlinearities be known *a priori* and that they be static, conditions seldom fulfilled in practice.

An alternative method for SISO systems that maintains the essential simplicity and small computational requirements of the STR has been presented by Papadoulis et al. (1987). Namely, the CSTC is a simple STC whose weight is tuned on-line, based on a measure of the plant/estimated model mismatch. The key idea is the incorporation of two parts in the control law. For poor parameters (large mismatch), a robust conservative controller becomes dominant. When the estimates improve to the point that the plant/model mismatch is negligible, a deadbeat controller becomes dominant and rapidly brings the system to the target steady state.

In this paper, the CSTC is extended for multiinput multioutput (MIMO) systems with multiple time delays. MIMO quadratic STC's have been developed first by Borisson (1979), Keviczky and Kumar (1981), and Koivo (1980), assuming the same time delay for each input-output pair. Subsequently, Tsiligiannis and Svoronos (1986, 1987) presented quadratic STC's applicable to systems with multiple delays with nontrivial cross couplings. All these controllers are based on constant weights penalizing the control variable with the associated problems discussed above.

Here the control weights are automatically adjusted by the algorithm on line so as to maintain the input and output variables bounded, despite any unmodeled dynamics, nonlinearities, and/or estimation error.

Model

A discrete time input-output representation of linear multivariable systems has the form

$$A'(q^{-1})y(t+1) = B'(q^{-1})u(t) + s' \quad (1)$$

where $y(t)$, $u(t)$ are the ℓ-dimensional input and output vectors, q^{-1} is the backward shift operator, and $A'(q^{-1})$, $B'(q^{-1})$ are polynomial matrices in q^{-1}. The ℓ-dimensional vector, s', represents deterministic disturbances which are either slowly varying or experiencing step changes.

As a matter of fact, it can be assumed without loss of generality that

$$A'(q^{-1}) = a'(q^{-1})I$$

with

$$a'(q^{-1}) = 1 + a'_1q^{-1} + \cdots + a'_\alpha q^{-\alpha}$$

and

$$(B'(q^{-1}))_{ij} = q^{-d_{ij}}b'_{ij}(q^{-1})$$

with

$$b'_{ij}(q^{-1}) = b'_{ij0} + b'_{ij1}q^{-1} + \cdots + b'_{ij\beta_{ij}}q^{-\beta_{ij}}$$

$$b'_{ij0} \neq 0 \quad \text{for every} \quad i,j$$

where d_{ij} is the time-delay between the i^{th} output and the j^{th} input.

In designing one-step-ahead controllers, a predictor form of Eq. 1 is needed. This step is not trivial in the presence of multiple delays and has not been resolved yet for the general case. However, there exist methods [(requiring *a priori* knowledge for the delay structure of $B'(q^{-1})$] which recast Eq. 1 into

$$A(q^{-1})\,w(t+d+1) = B(q^{-1})\,v(t) + s \quad (2)$$

with

$$A(q^{-1}) = \sum_{i=0}^{n} A_i q^{-i},\ A_0 = I \quad (3)$$

$$B(q^{-1}) = \sum_{i=0}^{m} B_i q^{-i},\ \det(B_0) \neq 0 \quad (4)$$

$$w(t) = q^{-k}K(q)y(t) \quad (5)$$

$$v(t) = q^{\mu}M(q^{-1})u(t) \quad (6)$$

and

$$d = k + \mu - 1$$

where $K(q)$, $M(q^{-1})$ are $\ell \times \ell$ polynomial matrices in q and q^{-1}, respectively, and k, μ are the maximum degrees in any element of $K(q)$, $M(q^{-1})$, respectively. The vector, $w(t)$, is known at time t, while $u(t)$ is determined from the present and previous $v(t)$. The matrices, $K(q)$, $M(q^{-1})$, are known and depend on the method used to obtain Eq. 2. It is required that $[q^{\mu}M(q^{-1})]^{-1}$ be a stable operator.

For example, if the left interactor matrix, $L(q)$, of model 1 is known (Goodwin and Sin, 1984, p. 132), then

$$M(q^{-1}) = I,\ \mu = 0$$

$$K(q) = L(q)$$

If the right interactor matrix, $R(q)$, is known (Tsiligiannis and Svoronos, 1986, then

$$K(q) = I \quad k = 0$$

$$M(q^{-1}) = R(q)^{-1}$$

Both of the above methods require the knowledge of the interactors, $L(q)$ or $R(q)$, and this can be restrictive in many cases. A method requiring the knowledge of only the time delays, d_{ij}, has been presented by Tsiligiannis and Svoronos (1987). According to this method,

$$K(q) = \text{diag}\,(q^{k_i}) \quad k = \max\,(k_i)$$

$$M(q) = \text{diag}\,(q^{\mu_i}) \quad \mu = \max\,(^{\mu}_{i})$$

and k_i and μ_i are found from d_{ij}'s by solving an integer programming problem. The method can treat large classes of systems with nontrivial cross couplings, but not the general case. An alternative method for a similar class of systems was proposed by Singh and Narendra (1984). The aim of all these methods is to obtain a parametrization model 2 of model 1 with $\det(B_0) \neq 0$ and $[q^{\mu}M(q^{-1})]^{-1}$ stable. Use in Eq. 2 of the identity,

$$I = F(q^{-1})A(q^{-1}) + q^{-d-1}G(q^{-1}) \quad (7)$$

where $F(q^{-1})$ is a polynomial matrix in q^{-1} of degree d, with leading coefficient I, and $G(q^{-1})$ is a polynomial matrix of degree $n-1$, results in the predictor form:

$$w(t+d+1) = C(q^{-1})\,w(t) + D(q^{-1})v(t) + r \quad (8)$$

where

$$C(q^{-1}) = G(q^{-1})$$

$$D(q^{-1}) = F(q^{-1})B(q^{-1}) \quad (D_0 = B_0)$$

and

$$r = F(1)s$$

Eq. 8 is equivalently written as

$$w(t + d + 1) = \theta x(t) = \theta_1 x_1(t) + B_0 v(t) \tag{9}$$

where θ is the model parameter matrix,

$$x^T(t) = [w^T(t) w^T(t-1) \cdots w^T(t-n) v^T(t) v^T(t-1) \cdots v^T(t-m'-d)\ 1] \tag{10}$$

and

$$x_1^T(t) = [w^T(t) w^T(t-1) \cdots w^T(t-n) v^T(t) v^T(t-1) \cdots v^T(t-m'-d)\ 1] \tag{11}$$

Estimates, $\hat{\theta}$, of the unknown model parameters, are obtained on line using a recursive parameter estimator. There are several algorithms that can be used (see, for example, Goodwin and Sin, 1984). It is strongly recommended, however, (see Fortescue et al., 1981) that an estimator that employs variable forgetting of past information (e.g., Fortescue et al., 1981) be used. Such an estimator is given in Papadoulis (1987).

The CSTC requires that $\hat{B}_0(t)$ be nonsingular. The time instants at which $\hat{B}_0(t)$ becomes singular form a set of measure zero. But even for those instants, it is shown in Goodwin and Sin (1984, p. 206) that a simple algorithm modification can remove the problem.

Multivariable Cautious Self-Tuning Controller

The CSTC is based on the following single-stage cost function:

$$J = |\hat{w}(t + d + 1|t) - w_r|^2 + [v(t) - v_r]^T Q(t)\,[v(t) - v_r] \tag{12}$$

$$\hat{w}(t + d + 1| = \hat{\theta}(t)x(t) = \hat{\theta}_1(t)x_1(t) + \hat{B}_0(t)v(t) \tag{13}$$

where $Q(t)$ is a positive semidefinite matrix and w_r is a reference signal given by

$$w_r = K(1)y_r$$

Also,

$$v_r = M(1)u_r$$

where u_r is the steady-state control vector corresponding to y_r (or an approximation to it). Eq. 12 differs from the cost function introduced in Clarke and Gawthrop (1975) in that $Q(t)$ is allowed to be time varying and is automatically adjusted on line.

Upon minimization with respect to $v(t)$, Eq. 12 results in the following expression for the control law:

$$[\hat{\theta}_1(t)x_1(t) - w_r] + [\hat{B}_0(t) + \hat{B}_0^{-T}(t)Q(t)]v(t) = \hat{B}_0^{-T}(t)Q(t)v_r \tag{14}$$

where the superscript, $-T$, denotes transposed inverse. If $Q(t) - 0$ at time t, the one-step-ahead dead-beat control policy is obtained.

$$v_{db}(t) = \hat{B}_0^{-1}(t)\,[w_r - \hat{\theta}_1(t)x_1(t)] \tag{15}$$

If $Q(t) = 0$ for every t, then Eq. 15 corresponds to the optimal dead-beat control law (minimum variance in the stochastic case).

Using Eqs. 14 and 15, we can express the cautious control law, $v(t)$, as a linear combination of the constant term, v_r, and the dead-beat control, v_{db}:

$$v(t) = H(t)v_r + [I - H(t)]v_{db}(t) \tag{16}$$

where

$$H(t) = [\hat{B}_0^T(t)\hat{B}_0(t) + Q(t)]^{-1}Q(t) \tag{17}$$

The v_{db} term represents a "fast" performing control law which, if based on the true process model, achieves $w(t + d + 1) = w_r$. But if it is based on a poor model, it might lead to poor performance. On the other hand, the term, v_r is model independent and drives the system outputs close to y_r with open-loop dynamics. It represents a "worst case" robust control law. The positive semidefinite matrix, $H(t)$, weighs the contributions of the "fast" v_{db} and the robust v_r terms in Eq. 16.

In what follows, we choose $H(t)$ to be a diagonal matrix, i.e.,

$$H(t) = \text{diag}\,[h_1(t), h_2(t), \ldots, h_\ell(t)] \tag{18}$$

with

$$0 \le h_i(t) < 1 \quad i = 0, \ldots, \ell$$

Note that since Eq. 17 corresponds to

$$Q(t) = \hat{B}_0^T(t)\hat{B}_0(t)H(t)\,[I - H(t)]^{-1}$$

$Q(t)$ is positive semidefinite for the choice of $H(t)$.

The structural simplicity of Eqs. 16 and 18 leads to an easier and more transparent way of tuning the algorithm, through $H(t)$ instead of $Q(t)$. In the case of large plant/model mismatch, a large value of $h_i(t)$, close to one, is appropriate. This choice would make the robust term dominant and weigh out the unreliable v_{db} contribution. When the estimated parameter matrix, $\hat{\theta}(t)$, and Eq. 13 provide an accurate description of the process, $h_i(t)$ should be taken close to zero, making v_{db} the dominant term.

A method for relating $h_i(t)$ to the plant/model mismatch is given next. The objective is to obtain a control law which is bounded in spite of modeling inaccuracy and estimation errors.

From Eq. 18 we obtain

$$v(t) - v_r = [I - H(t)]\,[v_{db}(t) - v_r] \tag{19}$$

Define

$$\tilde{v}(t) = v(t) - v_r = [\tilde{v}_1(t), \ldots, \tilde{v}_\ell(t)]^T$$

and

$$\tilde{v}_{db}(t) = v_{db}(t) - v_r = [\tilde{v}_{db1}(t), \ldots, \tilde{v}_{db\ell}(t)]^T$$

Then, from Eq. 19

$$\tilde{v}_i(t) = [1 - h_i(t)]\tilde{v}_{dbi}(t) \quad i = 1, \ldots, \ell$$

or

$$|\tilde{v}_i(t)| = [1 - h_i(t)]|\tilde{v}_{dbi}(t)| \quad i = 1, \ldots, \ell \tag{20}$$

The following bound is imposed on $|\tilde{v}_i(t)|$

$$|\tilde{v}_i(t)| \leq \frac{\delta_i}{\eta(t)} \quad i = 1, \ldots, \ell \tag{21}$$

where $\eta(t)$ is a measure of the plant/model mismatch.

A large $\eta(t)$ restricts the control, $v_i(t)$, close to v_{ri}, and effectively weighs out erroneous contributions from $v_{dbi}(t)$. As the model improves, $\eta(t)$ decreases, allowing larger control changes, and the control law achieves faster performance. The parameter, δ_i, is a scaling factor.

A choice for $h_i(t)$ that satisfies Eq. 21 is

$$h_i(t) = \frac{\eta(t)}{\dfrac{\delta_i}{|\tilde{v}_{dbi}(t)|} + \eta(t)} \tag{22}$$

Note that $h_i(t) = 0$ only for $\eta = 0$.

The measure of plant/model mismatch used here is the same as the one introduced in Papadoulis et al. (1987). It is related to the output prediction error, $\nu(t)$, by

$$\eta(t) = \lambda\eta(t - 1) + (1 - \lambda)|\nu(t)|; 0 < \lambda < 1 \tag{23}$$

$$\nu(t) = w(t) - \hat{w}(t|t - d - 1)$$

In Eq. 23, $\eta(t - 1)$ is substituted by a large value, $\bar{\eta}$, at sampling instants of set-point changes (since it is known that the locally valid parameters will change).

Note that the method presented in Papadoulis et al. (1987) for tuning the weight, $h(t)$ [SISO equivalent to $h_i(t)$], is based on the minimization of control law sensitivity to the prediction error. In the present paper, $h_i(t)$ is tuned so as to keep the control deviation bounded, with bound inversely proportional to a measure of the plant/model mismatch. For SISO systems, Eq. 22 reduces to a form very similar to that of $h(t)$ of the previous work, namely

$$h_i(t) = \frac{\eta(t)}{\delta_i|\hat{b}_0(t)|\gamma(t) + \eta(t)}; \quad \gamma(t) = \frac{1}{|w_r - \hat{\theta}_1(t)\, x_1(t)|}$$

which differs from $h(t)$ only in $\gamma(t)$.

Remark 1. As mentioned above, v_r is a constant input. In this case, the only information required about our system is steady-state information. Assuming that some extra limited dynamic information is also available (a first order plus delay model, for example), then an alternate CSTC is possible. In that case, v_r can be a nonadaptive conservatively tuned controller, e.g., a detuned PI or IMC controller. Note that the last two choices eliminate steady-state offset that might be present when approximate values for v_r are used and $h_i(t)$'s do not converge to zero. An application of such an alternate CSTC has been presented in Papadoulis et al. (1987).

The CSTC, in addition to the parameters that adjust the speed of forgetting (see Fortescue et al., 1981, or Papadoulis, 1987), requires off-line tuning of the parameters, λ, δ_i, and $\bar{\eta}$. Since its performance is satisfactory over a wide range of values for these parameters, tuning is not difficult. For λ, a value close to 0.9 is recommended. To tune δ_i and $\bar{\eta}$, one can use the dynamic response to input step changes (one for each input) to estimate the parameters of model 8 and hence obtain $\nu(t)$. The δ_i are then chosen so that the h_i drop close to zero when the process establishes a new steady state, while $\bar{\eta}$ is chosen so that the h_i rise to close to one, in response to a substantial step change.

To summarize, the CSTC consists of the estimator equations (for model 8), the inverse of Eq. 6, and Eqs. 5, 16, 15, 18, 22 and 23. It requires the following *a-priori* information:

- The matrices, $K(q)$ and $M(q^{-1})$. For many systems, including almost all two-dimensional, this requires only knowledge of the time delays (and for some time delays, e.g., those not involved in the determination of an interactor used, upper bounds are sufficient).
- Upper bounds in the orders of the polynomials of model 2
- Approximate input steady-state values. The more accurate these values are, the better, but values predicted from the steady-state gains at a nominal steady state will in many cases suffice.
- Dynamic data from input step responses, to aid in tuning the off-line parameters

Stability and Robustness of CSTC

The CSTC has been designed with nonideal applications in mind. It will be shown that, under assumptions satisfied by many nonlinear systems, the input and output variables remain bounded. First, the following definition is needed:

Definition 1. If there exists a finite P and a small positive ϵ such that for all t there exists a $t_1 \epsilon [t, t - P)$ with $|\nu(t_1)| > \epsilon$, then $\nu(t)$ will be referred to as "nondisappearing." If $\nu(t)$ is not "nondisappearing" it will be referred to as "disappearing."

Remark 2. Although it may seem that if $|\nu(t)|$ does not converge to zero it is "nondisappearing," this is not always the case. As an example, consider $\nu(t) = 1,0,1,0,0,1,0,0,0,1,0,0,0,0,1,0, \ldots$, which is "disappearing."

Theorem 1. Assume that: 1. The output prediction error $\nu(t)$ cannot be "disappearing" unless the output is bounded or unless the input is bounded; 2. the true process is bounded input-bounded output stable, i.e.,

$$\|y(t)\|_\infty \leq k_1 + k_2\|u(t)\|_\infty \tag{24}$$

and bounded output-bounded input stable, i.e.,

$$\|u(t)\|_\infty \leq k_3 + k_4\|y(t)\|_\infty \tag{25}$$

where $\|\cdot(t)\|_\infty$ denotes the ℓ_∞ norm truncated at time t, i.e.,

$$\|\cdot(t)\|_\infty = \max_{0 \leq \tau \leq t} |\cdot(\tau)|;$$

then, if the process is controlled by the CSTC, the input and output variables remain bounded.

Proof. Assume that both the input and the output are

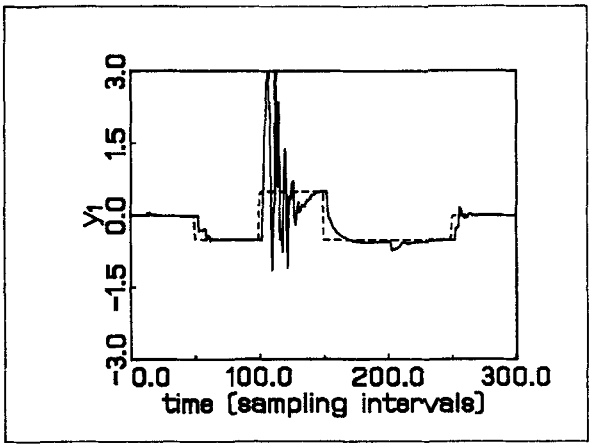

Figure 1. Dead-beat STC results.
———, y_1 vs. time; – – –, set-point

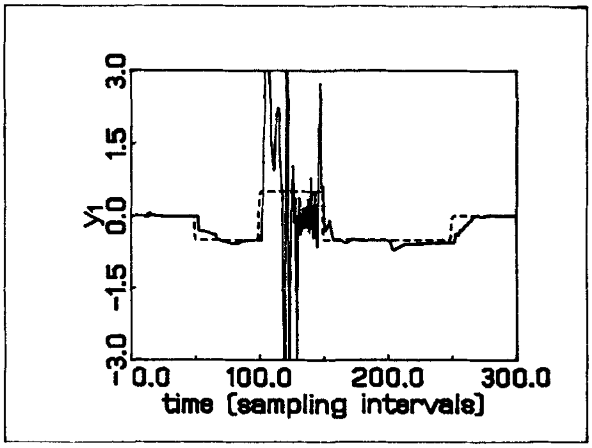

Figure 3. Extended horizon STC results.

unbounded. Since $\nu(t)$ is "nondisappearing" (assumption 1), it follows from Eq. 23 that $\eta(t)$ is bounded away from zero (one can easily show that $\eta(t) > (1 - \lambda)\ \lambda^{P}\epsilon$). Inequality 21 then implies that $v(t)$ is bounded, and since $[q^{\mu} M(q^{-1})]^{-1}$ is stable, it follows from Eq. 6 that $u(t)$ is bounded, thus contradicting the initial assumption. Therefore, either $u(t)$ or $y(t)$ is bounded. In the first case, Inequality 24 implies that $y(t)$ is bounded, while in the second case, Inequality 25 implies that $u(t)$ is bounded.

Remark 3. For nonlinear processes, since the model identified on line, Eq. 8, is linear, the output prediction error is a function of y and/or u, regardless of the values of the model parameters. Thus, assumption 1 will be satisfied for many nonlinear systems. Furthermore, it will be satisfied even for linear systems if the parameters converge to anything but the exact values.

Remark 4. For linear systems, Inequality 24 means that the system is open-loop stable and Inequality 25, that it is minimum phase.

CSTC Simulations for a Distillation Column Model

The performance of the CSTC is studied on a distillation column model described in Agarwal and Seborg (1987), and is compared to the performance of a dead-beat STC.

The model consists of two nonlinear ordinary differential equations:

$$\begin{aligned}
160\dot{y}_1(t') &= -y_1(t') + [-1.36 - 19.3y_1'(t') \\
&\quad + 0.0951y_2'(t')]u_1(t' - 650) \\
&\quad + [3.74 + 0.584y_1'(t') \\
&\quad + 3.50y_2'(t')]u_2(t' - 690) \\
200\dot{y}_2(t') &= -y_2(t') + [-0.125 - 0.237y_1'(t') \\
&\quad + 0.63y_2'(t')]u_1(t' - 640) \\
&\quad + [1.11 - 2.04y_1'(t') \\
&\quad - 0.229y_2'(t')]u_2(t' - 650) \\
y_1'(t') &= y_1(t')/[24 - y_1(t')] \\
y_2'(t') &= y_2(t')/[1 + y_2(t')] \qquad (26)
\end{aligned}$$

where the measured outputs, y_1 and y_2, are normalized deviation variables of the distillate and the bottoms streams mole fractions, respectively; the controls, u_1 and u_2, are normalized deviation variables of the reflux flow and the reboiler steam pressure, respectively; and t' is continuous time in seconds. For more information, the reader is referred to the paper by Agarwal and Seborg (1987). It is seen from Eq. 26 that the model is highly nonlinear and has various time delays. The process has strong

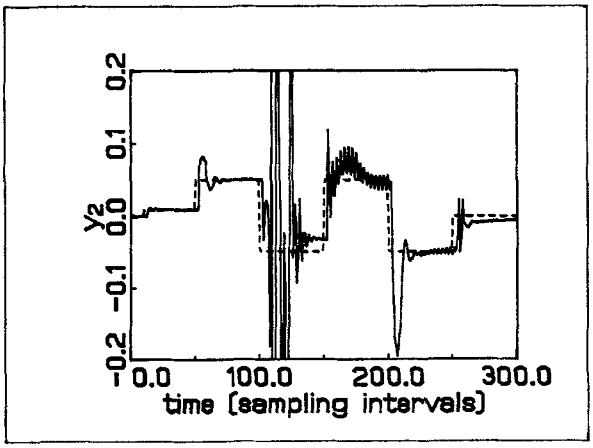

Figure 2. Dead-beat STC results.
y_2 (solid line) vs. time, dashed line – set-point

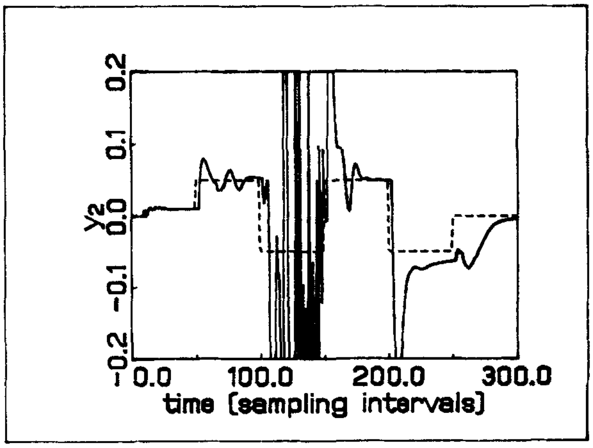

Figure 4. Extended horizon STC results.

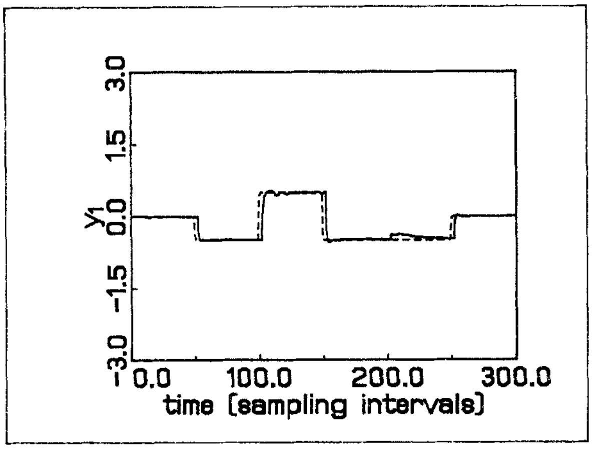

Figure 5. CSTC results.

parameters: $\delta_1 = \delta_2 = 0.01$; $\lambda = 0.9$; and $\bar{\eta} = 1$

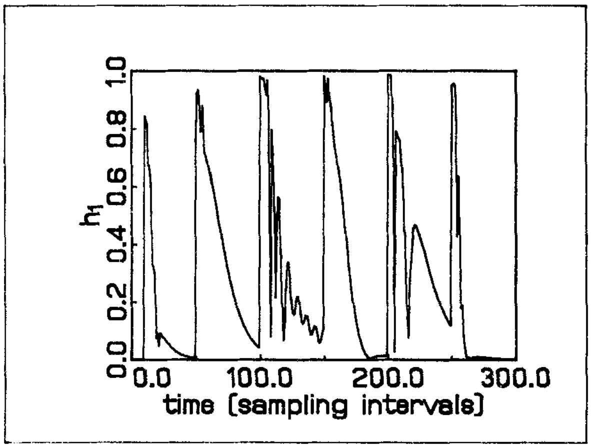

Figure 7. CSTC results.

interactions, as evidenced by a relative gain between y_1 and u_1 of 1.45 (at the nominal steady state, $y_1 = y_2 = u_1 = u_2 = 0$).

If the sampling period is chosen to be 300 s, adaptive control can be based on a linear discrete-time model of the form of Eq. 2 with constant parameters, $n = 2$, $m = 1$, $d = 2$, and variables, $w(t) = y(t)$, $v(t) = u(t)$. This is obtained either using the left interactor, in which case, $k = 3$, $K(q) = q^3 I$, $\mu = 0$, $M(q^{-1}) = I$, or using the right interactor, when $k = 0$, $K(q) = I$, $\mu = 3$, $M(q^{-1}) = q^{-3} I$. In both cases, $w(t) = y(t)$ and $v(t) = u(t)$, i.e., the same model results. This is because both interactors are diagonal, specifically, $L(q) = R(q) = q^3 I$.

A recursive least-squares estimator using the UDU covariance factorization (Bierman, 1977) and the variable forgetting factor of Fortescue et al. (1981), was employed for estimation of the parameters of the predictor form (Eq. 8) of the above described model (order of $C = n - 1 = 1$, order of $D = m + d = 3$).

First, the dynamic behavior of the dead-beat STC (Eq. 15) was tested for a series of simultaneous set-point changes on both y_1 and y_2. It can be seen that this linear STC follows the set points closely for small changes, but may perform poorly in response to large ones, as is evident from the oscillations seen in Figures 1 and 2. An adaptive controller of the GPC family was also tried, namely an extended-horizon algorithm of Ydstie and Liu (1984) (receding horizon, strategy two). Even with output horizon equal to ten sampling periods, it did not offer improvement over the dead-beat STC (see Figures 3 and 4). It should be remarked that all tested adaptive controllers employed the same forgetting factor tuning ($\Sigma_o = 10^{-2}$ and minimum $= 0.95$).

The performance of the cautious STC is shown in Figures 5 to 8. It is clear from Figures 5 and 6 that the excessive oscillations of the dead-beat STC have been eliminated with both outputs closely following the set points. Figures 7 and 8 depict the variation of the CSTC weights, $h_1(t)$ and $h_2(t)$. It is seen that, during initialization and after large set-point changes, h_1 and h_2 attain high values and then, as the model improves, decrease towards zero. For the reference inputs, v_r, approximate values were used, corresponding to substantial output steady-state offset (see Figures 9 and 10). Thus, an initial offset is present on the response of both outputs, diminishing rapidly as the estimated parameters improve and the weights converge to zero.

It is seen from this simulation example that the CSTC is a viable alternative for the adaptive control of nonlinear processes. It is simple, requiring minimum modeling information, is able to detune itself automatically in the presence of large plant/estimated model mismatch, and converges to an optimal controller whenever the estimated model provides an accurate dynamic description of the process.

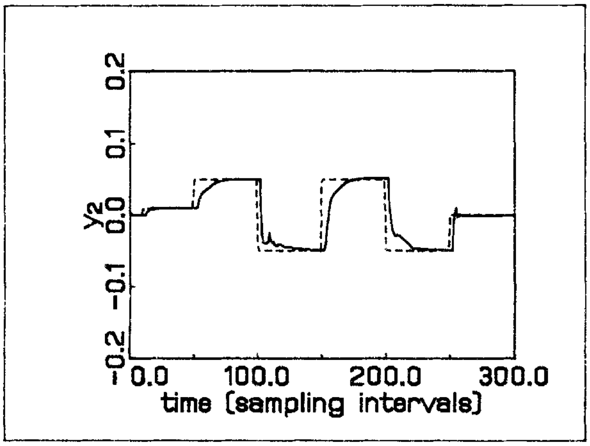

Figure 6. CSTC results.

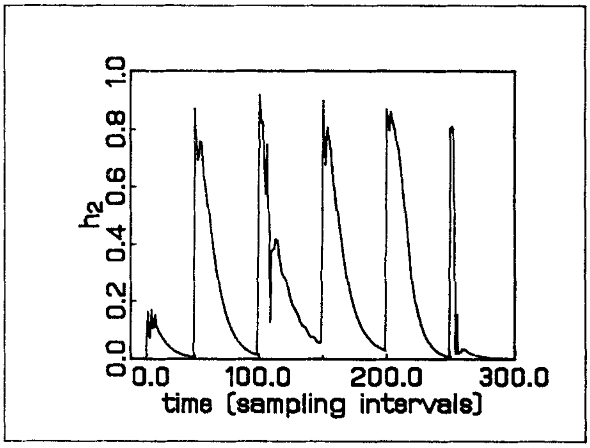

Figure 8. CSTC results.

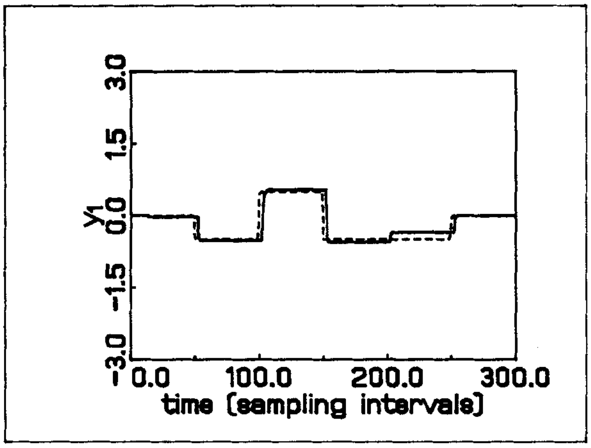

Figure 9. Results of control law, $u = v_r$.

Acknowledgment

The authors gratefully acknowledge financial support by the National Science Foundation under Grant No. CPE-8307973.

Notation

$A'(q^{-1})$ = polynomial matrix, Eq. 1
$a'(q^{-1})$ = scalar polynomial
a_i' = coefficients of $a'(q^{-1})$
$A(q^{-1})$ = polynomial matrix, Eq. 2
A_i = coefficient matrices of $A(q^{-1})$
$B'(q^{-1})$ = polynomial matrix, Eq. 1
$b_{ij}'(q^{-1})$ = elements of $B'(q^{-1})$
b_{ijk}' = coefficients of $b_{ij}'(q^{-1})$
$B(q^{-1})$ = polynomial matrix, Eq. 2
B_i = coefficient matrices of $B(q^{-1})$
$C(q^{-1})$ = polynomial matrix, Eq. 8
$D(q^{-1})$ = polynomial matrix, Eq. 8
d_{ij} = time delay between the i^{th} output and the j^{th} input
d = time delay parameter, Eq. 2
$F(q^{-1})$ = polynomial matrix, Eq. 7
$G(q^{-1})$ = polynomial matrix, Eq. 7
$H(t)$ = weight matrix, Eq. 17
$h_i(t)$ = diagonal elements of $H(t)$
I = identity matrix
k = output delay parameter, Eq. 5
$K(q)$ = polynomial matrix, Eq. 5
$L(q)$ = left interactor matrix
m = order of $B(q^{-1})$

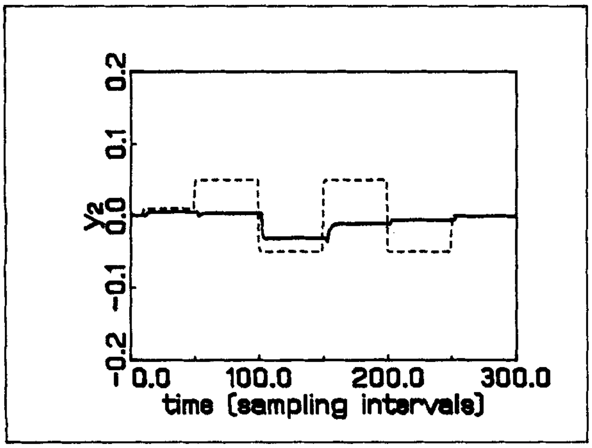

Figure 10. Results of control law, $u = r_r$.

m' = order of $D(q^{-1})$
$M(q^{-1})$ = polynomial matrix. Eq. 6
n = order of $A(q^{-1})$
P = finite time period, definition 1
q^{-1} = backward shift operator
$Q(t)$ = weight matrix, Eq. 12
$R(q)$ = right interactor matrix
r = parameter vector, Eq. 8
s' = parameter vector, Eq. 1
s = parameter vector, Eq. 2
t = discrete time
t' = continuous time
u = control vector
u_i = i^{th} control variable
v = vector of present and future inputs, Eq. 6
v_i = i^{th} component of v
w = vector of present and past measured outputs, Eq. 5
x = input/output vector, Eq. 10
x_1 = input/output vector, excluding $v(t)$, Eq. 11
y = measured output vector
y_i = i^{th} measured output
y_i' = function of y_i, Eq. 26

Greek letters

α = order of $a'(q^{-1})$
β_{ij} = order of $b_{ij}'(q^{-1})$
δ_i = scaling factor, Eq. 21
ϵ = small positive constant, definition 1
$\eta(t)$ = measure of plant/estimated model mismatch, Eq. 23
$\bar{\eta}$ = parameter associated with Eq. 23
θ = parameter matrix, Eq. 9
θ_1 = parameter matrix, Eq. 9
λ = filter coefficient, Eq. 23
μ = input delay parameter, Eq. 6
ν = output prediction error, Eq. 23

Superscripts

T = transpose
$-T$ = transposed inverse
$\hat{}$ = estimate
$\sim$ = deviation

Subscripts

db = dead beat
r = reference

Literature Cited

Agarwal, M., and D. E. Seborg, "A Multivariable Nonlinear Self-Tuning Controller," *AIChE J.*, **33**, 1379 (Aug., 1987).

Allidina, A. Y., and F. M. Hughes, "Generalized Self-Tuning Controller with Pole Assignment," *IEE Proc. Pt. D.*, **127**, 13 (1980).

Bierman, G. C., *Factorization Methods for Discrete Sequential Estimation*, Academic Press, New York (1977).

Borisson, U., "Self-Tuning Regulators for a Class of Multivariable Systems," *Automatica*, **15**, 209 (1979).

Clarke, D. W., and P. J. Gawthrop, "Self-Tuning Controller," *IEE Proc. Pt. D.*, **122**, 929 (1975).

Clarke, D. W., C. Mohtadi, and P. S. Tuffs, "Generalized Predictive Control: I. The Basic Algorithm," *Automatica*, **32**, 137 (1987a).

———, "Generalized Predictive Control: II. Extentions and Interpretations," *Automatica*, **32**, 149 (1987b).

Dumont, G. A., "On the Use of Adaptive Control in the Process Industries," Proc. 3rd Int. Conf. on Chem. Proc. Contr., Asilomar, CA, 467 (1986).

Fortescue, T. R., L. S. Kershenbaum, and B. E. Ydstie, "Implementation of Self-Tuning Regulators with Variable Forgetting Factors," *Automatica*, **17**, 831 (1981).

Gawthrop, P. J., and K. W. Lim, "Robustness of Self-Tuning Controllers," *IEE Proc. Pt. D.*, **123**, 21 (1982).

Golden, M. P., and B. E. Ydstie, "Nonlinear Model Control Revisited," Proc. Amer. Contr. Conf., Boston, MA, 1553 (1985).

Goodwin, G. C., and K. S. Sin, *Adaptive Filtering Prediction and Control*, Prentice-Hall, Englewood Cliffs, NJ (1984).
Gustafsson, T. K., "Adaptive Methods for Linear and Nonlinear Control of pH," 187th ACS Nat. Meeting, St. Louis, MO (1984).
Keviczky, L., and K. S. P. Kumar, "Multivariable Self-Tuning Regulator with Generalized Cost-Function," *Int. J. Contr.*, **33**, 213 (1981).
Koivo, H. N., "A Multivariable Self-Tuning Controller," *Automatica*, **16**, 351 (1980).
Latawiec, K., and M. Chyra, "On Low Frequency and Long-Run Effects in Self-Tuning Control," *Automatica*, **19**, 419 (1983).
Papadoulis, A. V., C. A. Tsiligiannis, and S. A. Svoronos, "A Cautious Self-Tuning Controller for Chemical Processes," *AIChE J.*, **33**, 401 (Mar., 1987).
Papadoulis, A. V., "Towards Stable Adaptive Control for Nonlinear System," PhD Diss., University of Florida (1987).
Singh, R. P., and K. W. Narendra, "Prior Information in the Design of Multivariable Adaptive Controllers," *IEEE Trans. Aut. Contr.*, **AC-29**, 1108 (1984).
Song, H. K., D. G. Fisher, and S. L. Shah, "Experimental Evaluation of Adaptive Control Methods on a Pilot Plant Evaporator," Proc. Amer. Control Conf., San Diego, CA, 1843 (1984).
Toivonen, H. T., "Variance Constrained Self-Tuning Control," *Automatica*, **19**, 415 (1983).
Tsiligiannis, C. A., and S. A. Svoronos, "Multivariable Self-Tuning Control via the Right Interactor Matrix," *IEEE Trans. Aut. Contr.*, **AC-31**, 987 (1986).
———, "An Adaptive Controller for Multivariable Delayed Systems," *Int. J. Contr.*, **45**, 2059 (1987).
Ydstie, B. E., and L. K. Liu, "Single- and Multi-Variable Control with Extended Prediction Horizons," Proc. Amer. Contr. Conf., San Diego, CA, 1303 (1984).
Ydstie, B. E., L. S. Kershenbaum, and R. W. H. Sargent, "Theory and Application of an Extended Horizon Self-Tuning Controller," *AIChE J.*, **31**, 771 (May, 1985).

Manuscript received June 20, 1988, and revision received June 2, 1989.